FitzGerald Relativity, Part One: Foundations

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1 Introduction

1.1 Scientific Theory

The ancients created myths populated with fantastic entities, extraordinarly persons, and gods. These memorable stories created a language for describing why things are as they are, and they guided expectation. This is what theories do, too. They create an idiom for discussion, (the normative or prescriptive aspect of theory,) and prompt more or less useful expectation, (the positive, descriptive or predictive aspect of theory.) Some theory, like music theory, is almost purely normative. Mathematical theories, like group theory, set theory, and graph theory, are purely normative. Newton's second law was partly normative in that it created a way to quanitfy force in terms of existing metrics for time, distance and mass. Most scientific theory is both normative and positive. The myths of today include scientific theories. Scientific theories are stories that model reality in our thought, but they are not the reality itself.

1.2 Scientific Standards

Scientific theory is supposed to meet certain requirements: logical self-consistency, usefulness, testability. How do these criteria apply to normative and positive parts of a theory?

Intelligibility and logical self-consistency are required of the normative part of a theory. These are supported by a principle, Ockham's razor, associated with William of Ockham (c. 1287–1347), "Entities must not be multiplied beyond necessity," meaning the number of entities created or assumptions made is best kept to a minimum. Humorously stated, a theory should be as simple as possible but no simpler.

Testability, sometimes called falsifiability, pertains to the positive part of theory as clarified by Karl Popper (1902 – 1994), eminent philosopher of science.

1. It is easy to obtain confirmations, or verifications, for nearly every theory — if we look for confirmations.
2. Confirmations should count only if they are the result of risky predictions; that is to say, if, unenlightened by the theory in question, we should have expected an event which was incompatible with the theory — an event which would have refuted the theory.
3. Every "good" scientific theory is a prohibition: it forbids certain things to happen. The more a theory forbids, the better it is.

4. A theory which is not refutable by any conceivable event is non-scientific. Irrefutability is not a virtue of a theory (as people often think) but a vice.
5. Every genuine test of a theory is an attempt to falsify it, or to refute it. Testability is falsifiability; but there are degrees of testability: some theories are more testable, more exposed to refutation, than others; they take, as it were, greater risks.
6. Confirming evidence should not count except when it is the result of a genuine test of the theory; and this means that it can be presented as a serious but unsuccessful attempt to falsify the theory. (I now speak in such cases of "corroborating evidence.")
7. Some genuinely testable theories, when found to be false, are still upheld by their admirers — for example by introducing ad hoc some auxiliary assumption, or by reinterpreting the theory ad hoc in such a way that it escapes refutation. Such a procedure is always possible, but it rescues the theory from refutation only at the price of destroying, or at least lowering, its scientific status. (I later described such a rescuing operation as a "conventionalist twist" or a "conventionalist stratagem.")

> One can sum up all this by saying that the criterion of the scientific status of a theory is its falsifiability, or refutability, or testability.[1]

Of course, no one considers refuting music theory.

The “conventionalist strategem” to handle falsification is a not uncommon stop-gap when a satisfactory replacement theory has not yet been crafted.

Usefulness is realized from the conjunction of normative and positive parts. The normative aspect of a relativity theory must provide a consistent metrical framework for description, (positive part,) of events in the physical world, in particular, of the time and place of any event. It must provide a foundation for precise, lucid description of physical phenomena and of the physical laws and theories we construct to explain them. Its applicability must extend to any frame of reference.

Alternative theories may co-exist. A relativity theory must be judged first on whether it is self-consistent, then on whether its positive elements have withstood all tests, only then on usefulness or ease of use. A novel theory of relativity has an extra burden inasmuch as novelty compromises ease of use.

[1] Karl Popper, Conjectures and Refutations, London: Routledge and Keagan Paul, 1963, pp. 33-39; from Theodore Schick, ed., Readings in the Philosophy of Science, Mountain View, CA: Mayfield Publishing Company, 2000, pp. 9-13.

1.3 Scientific Revolution

What happens when a theory is falsified? This question was brilliantly answered by Thomas Kuhn in his book *The Structure of Scientific Revolutions.* The surprising answer, (with only slight hyperbole,) is, “nothing.” Kuhn’s study of historical cases reveals that normal science tends to be tightly focused on working within the prevailing paradigm. Evidence that a theory is false tends to be ignored, dismissed as flawed, dismissed as paradoxical, rationalized with a conventionalist twist and so forth. Most efforts are aimed at confirming the prevailing theory, evaluating parameters of the theory, and finding new applications of the theory. When a revolutionary new theory replaces an old theory it is largely the dying off of the old guard as a younger generation embraces the new theory. Perhaps, having a replacement theory ready to go when the old theory is discredited would facilitate a break from that unfortunate pattern.

Special relativity theory has vulnerabilities. One, a corollary of special relativity, that one way speed of light is the same in every direction for every inertial frame, can be experimentally tested.[2]

FitzGerald relativity is herein proposed to be the successor to Einstein’s special theory of relativity if the one way speed of light is found to be the same in every direction only for a unique rest frame of reference. Another alternative theory, featuring an absolute frame of reference, might be based on the work of Lorentz and Larmor.

1.4 Introduction to FitzGerald Relativity

It is well known that Heinrich Anton Lorentz acknowledged the priority of George Francis FitzGerald in suggesting the Michelson Morley experiment of 1887 might be explained by contraction of material bodies due to their velocity relative to the luminiferous ether. The transformation equations of Albert Einstein’s special relativity (1905) are in consequence sometimes called the Lorentz-FitzGerald transformations. Yet, these three held different concepts.

FitzGerald conjectured in 1889:

> . . . that almost the only hypothesis that can reconcile this . . . is that the length of material bodies changes, according as they are moving through the ether or across it, by an amount depending on the square of the ratio of their velocity to that of light. We know that electric forces are affected by the motion of the electrified bodies relative to the ether, and it seems

[2] Wallace, D. B., "A Revealing Test of the Compatibility of Special Relativity Postulates"

> a not improbable supposition that the molecular forces are affected by the motion, and that the size of a body alters consequently.[3]

This remarkable conjecture anticipated recognition that chemical bonds are electromagnetic in nature and suggested that length contraction consequently occurs across as well as along the direction of motion. It thus uniquely offers a physical explanation of Michelson's and Morley's null result.

The electromagnetic field is not a material substance, but the phenomena it models are evident. According to Maxwell's theory of electromagnetism, disturbances in the electromagnetic field are propagated isotropically at a constant speed c in free space. The field, not some ethereal substance, is the conceived medium of this propagation. Speculation about a dragged or deformed luminiferous ether was triggered by the surprising outcome of the Michelson Morley experiment. Michelson expected that the round trip of light between two points on a stone slab of presumed stable dimensions, would take longer if the stone slab were moving relative to the ether, by the factor $1/(1-v^2/c^2)$ if the points were aligned parallel to their velocity $\overrightarrow{v}$, and by $1/\sqrt{1-v^2/c^2}$ if aligned perpendicular to the velocity. However, he observed no discernable difference.

The purport of FitzGerald's conjecture was that the two points were not a fixed distance apart as Michelson had supposed; rather, that the forces holding a material body together were electromagnetic, (a novel idea in itself,) and governed the material dimensions of the stone slab causing contraction by the factor $1-v^2/c^2$ along the direction of motion and by $\sqrt{1-v^2/c^2}$ perpendicular to it; thus cancelling the expected change in round trip time.

In their attempts to account for the Michelson Morley null result, both Lorentz and FitzGerald related the length contraction of their conjectures to velocity relative to an absolute frame of reference, the luminiferous ether, characterized by isotropy of one way light speed. Lorentz, unlike Einstein, was not committed to zero transverse contraction, but noted that any deformation related to velocity must have a longitudinal to transverse ratio of $\sqrt{1-v^2/c^2}$.

Einstein rejected the notion of a luminiferous ether. He declared "absolute rest" meaningless and considered velocity to be strictly relative. Einstein, by his own testimony, was unfamiliar with the Michelson Morley experiment when he wrote his special relativity paper. He envisioned trying to measure the speed of light using clocks, though clocks of that day were nowhere near stable and accurate enough for the purpose. His special relativity was a speculation without supporting experimental evidence.

[3]FitzGerald, G. F., "The Ether and the Earth's Atmosphere," Science v. XIII No. 328, p. 390, 1889

FitzGerald died in 1901, prior to the advent of special relativity. He did not include equations in his brief conjecture. From his words, however, a different set of transformation equations is easily constructed. From these FitzGerald equations an entirely new theory of relativity unfolds, no less empirically successful, more intuitive, free of ambiguities and paradoxes and incorporating the notion of absolute rest. Detailed description of this FitzGerald relativity is the subject of this series.

2 FitzGerald Coordinate Transformations

2.1 Events and Space-Time Coordinates

In keeping with the instincts of FitzGerald and Lorentz, only one inertial frame of reference, called the rest frame, will be deemed absolutely stationary so that light speed is isotropic, i. e. the same in every direction. Throughout this paper, reference will be made to local frames of reference, each being fully specified by its origin and its constant velocity relative to the rest frame. The absolute velocity of a local frame will usually be given as the ratio of its velocity to the speed of light, $\overrightarrow{\beta} = \frac{\vec{v}}{c}$. Local frames differing in choice of origin are yet the same inertial frame if the coordinate origins have the same velocity relative to the rest frame.

Time is understood to be one thing, not a different thing in each frame of reference. All clocks are to be synchronized in the rest frame. The use of local time synchronization, based on the assumption that light signals between the local origin and the clock take the same time in each direction, is deprecated as a fiction in all but the rest frame where it is strictly true; however, the likelihood that the practice will continue compels inclusion of a transformation between local time and rest frame time. Current international time standards are synchronized in the earth center inertial frame because earth rotation puts clocks around the world in different inertial frames; they could as well use the rest frame rather than the earth center frame.

Discussions of light and time will be idealized with light speed always c relative to the rest frame, and clock rate the same for all clocks regardless of motion or position.

As a point has three spatial coordinates, (x, y, z), an event has four coordinates, (x, y, z, t), three of space and one of time. The spatial coordinate values are frame of reference specific because spatial coordinates will be in local length units. The coordinates of an event E are either given as absolute (E_0 relative to the rest frame,) or local (E_β local length with rest frame time,) but may for special purposes be either fully local including local time ($E_{\beta,\text{local}}$) or rest frame metrics relative to a local origin ($E_{\beta,0}$ subscript order being "frame-of-reference, metric.")

2.2 Required Alignment of Coordinates

Lorentz equations of special relativity and FitzGerald equations both transform space-time coordinates of an event relative to one inertial coordinate system into the space-time coordinates of the same event relative to another. Both require the x-axes of the two systems to coincide and the other axes to be parallel. Only the FitzGerald equations also require that one frame be the rest frame and that the other frame use rest frame time rather than local time. "Rest frame" is deemed meaningless in special relativity. Both Lorentz and FitzGerald equations require the origins to coincide at time zero; this puts a constraint on origin choice for Lorentz equations,[4] but not for FitzGerald equations.

In special relativity, the velocity variable of the Lorentz equations is the velocity of one inertial frame relative to another. The length contraction and time dilation are held to be virtual and frame of reference dependent. The same equations serve as the inverse transformation. Thus special relativity denies the uniqueness of the rest frame. The normative definitions included in special relativity preclude identification of an absolute rest frame.

The velocity variable v_0 of the FitzGerald equations is velocity in the positive x-direction relative to absolute rest; in FitzGerald relativity it is possible to deduce absolute velocity from empirical tests, (to be addressed in part three.) Length contraction is understood to be actual contraction of condensed matter. The contraction of moving solid length standards produces a virtual lengthening of spatial distance as compared to rest frame measure. All frames share rest frame time so there is no time dilation.[5] The reverse transformation is distinct from the forward transformation.[6] The frame of reference of a variable will be indicated by a subscript, usually the frame's velocity as a fraction of light speed, e. g. ϕ_β, with a subscript zero for the absolute rest frame, e. g. ϕ_0.

2.3 Equation Summary

The name of a vector will represent the magnitude of the vector unless clearly shown as a vector, e. g. $\overrightarrow{\beta}$.

Here for comparison are the equations.

[4]When local time is used, the local origin must be a point where local time coincides with rest frame time.

[5]Clock rate changes are not conflated with time rate changes.

[6]Rest frame is considered un-transformed, so "forward" is from rest frame to moving frame, and "reverse" restores rest frame.

Lorentz Equations		FitzGerald Equations		
$\beta \equiv \frac{v}{c}$		$\beta \equiv \frac{v_0}{c}$		
$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$		$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$		
	Forward		Reverse	
$t_B = \gamma\left(t_A - \frac{vx_A}{c^2}\right)$	$t = t$		$t = t$	
$x_B = \gamma(x_A - vt_A)$	$x_\beta = \gamma^2(x_0 - \beta ct)$	(22)	$x_0 = \frac{x_\beta}{\gamma^2} + \beta ct$	(25)
$y_B = y_A$	$y_\beta = \gamma y_0$	(23)	$y_0 = \frac{y_\beta}{\gamma}$	(26)
$z_B = z_A$	$z_\beta = \gamma z_0$	(24)	$z_0 = \frac{z_\beta}{\gamma}$	(27)
Local time:	$t_\beta = \gamma^2\left(t_0 - \frac{\beta x_0}{c}\right)$	(29)	$t_0 = t_\beta + \frac{\beta x_\beta}{c}$	(30)
Good to know:	$\gamma^2\beta^2 = \gamma^2 - 1$		$0 \leq \beta < 1 \leq \gamma$	
Trig Functions	$\tan\phi_\beta = \frac{\tan\phi_0}{\gamma}$	(31)	$\tan\phi_0 = \gamma\tan\phi_\beta$	(32)
of Angle	$\sin\phi_\beta = \frac{\sin\phi_0}{\gamma\sqrt{1-\beta^2\sin^2\phi_0}}$	(33)	$\sin\phi_0 = \frac{\sin\phi_\beta}{\sqrt{1-\beta^2+\beta^2\sin^2\phi_\beta}}$	(34)
from $\overrightarrow{\beta}$	$\cos\phi_\beta = \frac{\cos\phi_0}{\sqrt{1-\beta^2+\beta^2\cos^2\phi_0}}$	(35)	$\cos\phi_0 = \frac{\cos\phi_\beta}{\gamma\sqrt{1-\beta^2\cos^2\phi_\beta}}$	(36)

The alert reader will have noticed that, for local time and for spatial coordinates, the right sides of the Lorentz equations, if multiplied by γ, yield the right sides of the forward FitzGerald equations, and for all but the x coordinate, if divided by γ, yield the right sides of the reverse FitzGerald equations. The use of rest frame time rather than local time is responsible for the x coordinate exception.

2.4 Transformation Matrices

Transformation of position vectors can be effected with matrix operators.

$$\mathbf{p}_0\mathbf{F} - \gamma^2\mathbf{q} = \mathbf{p}_\beta \tag{1}$$

$$\mathbf{p}_\beta\mathbf{G} + \mathbf{q} = \mathbf{p}_0 \tag{2}$$

where

$$\mathbf{p}_0 = \begin{bmatrix} x_0 & y_0 & z_0 \end{bmatrix} \tag{3}$$

$$\mathbf{q} = \begin{bmatrix} \beta ct & 0 & 0 \end{bmatrix} \tag{4}$$

$$\mathbf{p}_\beta = \begin{bmatrix} x_\beta & y_\beta & z_\beta \end{bmatrix} \tag{5}$$

$$\mathbf{F} = \begin{bmatrix} \gamma^2 & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma \end{bmatrix} \tag{6}$$

$$\mathbf{G} = \begin{bmatrix} \frac{1}{\gamma^2} & 0 & 0 \\ 0 & \frac{1}{\gamma} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{bmatrix} \tag{7}$$

$$\mathbf{F}\mathbf{p}_0 - \gamma^2\mathbf{q} = \mathbf{p}_\beta \tag{8}$$

$$\mathbf{G}\mathbf{p}_\beta + \mathbf{q} = \mathbf{p}_0 \tag{9}$$

where

$$\mathbf{p}_0 = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \tag{10}$$

$$\mathbf{q} = \begin{pmatrix} \beta ct \\ 0 \\ 0 \end{pmatrix} \tag{11}$$

$$\mathbf{p}_\beta = \begin{pmatrix} x_\beta \\ y_\beta \\ z_\beta \end{pmatrix} \tag{12}$$

$$\mathbf{F} = \begin{pmatrix} \gamma^2 & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix} \tag{13}$$

$$\mathbf{G} = \begin{pmatrix} \frac{1}{\gamma^2} & 0 & 0 \\ 0 & \frac{1}{\gamma} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{pmatrix} \tag{14}$$

2.5 Equation Derivations

2.5.1 A Round Trip of Light

To begin our derivation of the relation of frame dependent quantities, see figure 1, representing the round trip of a light signal from point P to Q and back, as P and Q move at constant velocity $v_0 = \beta c$ relative to the rest frame. Although P and Q are moving in the rest frame, they are fixed relative to each other.

We take A and P as the origins of stationary and moving frames, respectively, coinciding at $t = 0$. We let the x-axis be parallel to the velocity. We let the z-coordinate go unexpressed as it is zero throughout.

The rest frame time scale is used everywhere, as if clocks have been synchronized using light or radio signals understood to have velocity c relative to the rest frame.

The light signal, to begin its round trip, originates from P when P is at point A, our space-time origin. The light signal reaches Q at time $t = T_1$ when Q and P are at points B and D respectively. The signal, being reflected at B, returns to P at $t = T_1 + T_2$ when P is at point C. The points A, B, C, and D are fixed points in the rest frame marking the locations of these events. Light path lengths are: from A to B, cT_1, and from B to C, cT_2.

The rest frame measure of angle $\angle CDB$ between the velocity $\overrightarrow{v_0} = \overrightarrow{\beta} c$ and the direction P to Q is ϕ_0. The rest frame distance between points P and Q, (same as the distance between D and B,) is d_0. Path lengths for the moving point P are from A to D, $\beta c T_1$, and from D to C, $\beta c T_2$. All times, distances and angles are relative to the rest frame.

We explore the relatonship of these quantities in pursuing our ultimate purpose to derive the moving frame distance d_β.

The cosine law and quadratic formula yield solutions in variables d_0, βc, and ϕ_0 for T_1 and T_2 in the triangles $\triangle ADB$ and $\triangle CDB$ respectively. First, we find T_1 of triangle $\triangle ADB$ in equations (15) through (17) using the law of cosines.

$$c^2T_1^2 = \beta^2c^2T_1^2 + d_0^2 + 2\beta c T_1 d_0 \cos\phi_0 \tag{15}$$

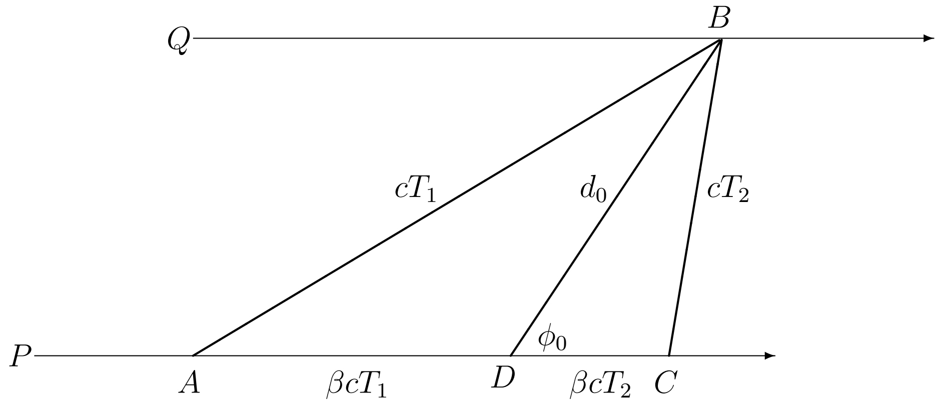
We write (15) in standard quadratic form.

$$(1-\beta^2)c^2T_1^2 - 2\beta c d_0 \cos\phi_0 T_1 - d_0^2 = 0 \tag{16}$$

We solve for T_1 using the quadratic formula.

$$T_1 = \frac{d_0\left(\beta\cos\phi_0 + \sqrt{1-\beta^2\sin^2\phi_0}\right)}{c(1-\beta^2)} \tag{17}$$

Figure 1



Now, we find T_2 of triangle $\triangle CDB$ in equations (18) through (20).

$$c^2T_2^2 = \beta^2c^2T_2^2 + d_0^2 - 2\beta cT_2d_0\cos\phi_0 \tag{18}$$

We write (18) in standard quadratic form.

$$(1-\beta^2)c^2T_2^2 + 2\beta cd_0\cos\phi_0 T_2 - d_0^2 = 0 \tag{19}$$

We solve for T_2 using the quadratic formula.

$$T_2 = \frac{d_0\left(-\beta\cos\phi_0 + \sqrt{1-\beta^2\sin^2\phi_0}\right)}{c(1-\beta^2)} \tag{20}$$

2.5.2 Moving Length

In the proper frame of P and Q, the distance from P to Q is judged by the round trip time of the light signal, $d_\beta = \frac{c(T_1+T_2)}{2}$.

$$d_\beta = \frac{c}{2}\left(\frac{d_0\left(\beta\cos\phi_0 + \sqrt{1-\beta^2\sin^2\phi_0}\right)}{c(1-\beta^2)} + \frac{d_0\left(-\beta\cos\phi_0 + \sqrt{1-\beta^2\sin^2\phi_0}\right)}{c(1-\beta^2)}\right)$$

which simplifies to

$$d_\beta = \gamma^2 d_0\sqrt{1-\beta^2\sin^2\phi_0} \tag{21}$$

The spatial distance appears greater in the moving frame because the local measuring sticks are contracted.

2.5.3 Spatial Coordinate Conversion

The rest frame coordinates of Q are $(\beta ct + d_0 \cos\phi_0, d_0 \sin\phi_0)$. For length parallel to the velocity, the special case $\phi_0 = 0$, our moving length equation yields the forward transformation of the x-coordinate.

$$x_\beta = \gamma^2(x_0 - \beta ct) \tag{22}$$

Take note that when transforming the difference of x-coordinates at a fixed time, the βct terms cancel, so $\Delta x_\beta = \gamma^2 \Delta x_0$.

For the perpendicular case, $\phi_0 = \frac{\pi}{2}$, we have our forward transformation of the y-coordinate.

$$y_\beta = \gamma y_0 \tag{23}$$

By symmetry, the forward transformation of the z-coordinate is

$$z_\beta = \gamma z_0 \tag{24}$$

Now we can solve the equations (22) through (24) to find the reverse transformations.

$$x_0 = \frac{x_\beta}{\gamma^2} + \beta ct \tag{25}$$

$$y_0 = \frac{y_\beta}{\gamma} \tag{26}$$

$$z_0 = \frac{z_\beta}{\gamma} \tag{27}$$

2.5.4 Reverse Length Conversion

$$\begin{aligned} d_0 &= \sqrt{x_0^2 + y_0^2 + z_0^2} \\ &= \sqrt{\frac{x_\beta^2}{\gamma^4} + \frac{y_\beta^2 + z_\beta^2}{\gamma^2}} \\ &= \sqrt{\frac{d_\beta^2 \cos^2\phi_\beta}{\gamma^4} + \frac{d_\beta^2 \sin^2\phi_\beta}{\gamma^2}} \\ &= \frac{d_\beta}{\gamma}\sqrt{(1-\beta^2)\cos^2\phi_\beta + \sin^2\phi_\beta} \end{aligned}$$

$$d_0 = \frac{d_\beta}{\gamma}\sqrt{1 - \beta^2 \cos^2\phi_\beta} \tag{28}$$

2.5.5 Local Time

FitzGerald relativity is normative in the matter of time. It does not make claims about the physics of clocks; rather, it prescribes what clocks should do. One time scale for all frames is fundamental to FitzGerald relativity.

We now consider the deprecated use of local time and how it relates to the standard rest frame time of FitzGerald relativity. We must heed the requirement, familiar in special relativity, that the origins of the two frames be coincident and synchronized at time zero. The synchronizations are coordinate dependent, but the length of the time unit is to be the same in all frames so the moving origin will remain synchronized with the rest frame.

Referring again to figure 1, the reflection at B occurs at $t_0 = T_1$, but in the frame of P and Q local time of the reflection is taken to be $t_\beta = \frac{T_1+T_2}{2}$.

$$\begin{aligned}
t_\beta - t_0 &= \frac{T_1 + T_2}{2} - T_1 \\
&= \frac{T_2 - T_1}{2} \\
&= \frac{\left(\frac{d_0\left(-\beta\cos\phi_0 + \sqrt{1-\beta^2\sin^2\phi_0}\right)}{c(1-\beta^2)}\right) - \left(\frac{d_0\left(\beta\cos\phi_0 + \sqrt{1-\beta^2\sin^2\phi_0}\right)}{c(1-\beta^2)}\right)}{2} \\
&= \frac{-d_0\beta\cos\phi_0}{c(1-\beta^2)}
\end{aligned}$$

$d_0\cos\phi_0 = (x_0 - \beta c t_0)$ and $\frac{1}{1-\beta^2} = \gamma^2$ so this becomes,

$$t_\beta - t_0 = \frac{-\beta\gamma^2(x_0 - \beta c t_0)}{c}$$

$$t_\beta = \gamma^2\left(t_0 - \frac{\beta x_0}{c}\right) \tag{29}$$

At the origin of the moving frame with $x_0 = \beta c t_0$ this reduces to $t_\beta = t_0$, as required.

The reverse transformation is

$$t_0 = \frac{t_\beta}{\gamma^2} + \frac{\beta x_0}{c}$$

Substituting for x_0,

$$t_0 = \frac{t_\beta}{\gamma^2} + \frac{\beta\left(\frac{x_\beta}{\gamma^2} + \beta c t_0\right)}{c}$$

Re-solving for t_0,

$$t_0 = t_\beta + \frac{\beta x_\beta}{c} \tag{30}$$

These time transformations differ from the Lorentz time transformations by the factor γ.

2.5.6 Angle Measure

By applying coordinate conversion to the trigonometric functions of ϕ, a stable[7] angle with one ray parallel to the x-axis, we learn the relation of rest and moving frame measure of angles.

Tangent forward,

$$\tan\phi_\beta = \frac{\Delta y_\beta}{\Delta x_\beta} = \frac{\tan\phi_0}{\gamma} \tag{31}$$

and reverse,

$$\tan\phi_0 = \gamma\tan\phi_\beta \tag{32}$$

Sine forward,

$$\sin\phi_\beta = \frac{\Delta y_\beta}{\Delta d_\beta} = \frac{\gamma\Delta y_0}{\gamma^2\Delta d_0\sqrt{1-\beta^2\sin^2\phi_0}}$$

$$\sin\phi_\beta = \frac{\sin\phi_0}{\gamma\sqrt{1-\beta^2\sin^2\phi_0}} \tag{33}$$

and reverse,

$$\sin\phi_0 = \frac{y_0}{d_0} = \frac{\sin\phi_\beta}{\sqrt{1-\beta^2\cos^2\phi_\beta}}$$

or written with sine only

$$\sin\phi_0 = \frac{\sin\phi_\beta}{\sqrt{1-\beta^2+\beta^2\sin^2\phi_\beta}} \tag{34}$$

Cosine forward,

$$\cos\phi_\beta = \frac{\Delta x_\beta}{\Delta d_\beta} = \frac{\gamma^2\Delta x_0}{\gamma^2\Delta d_0\sqrt{1-\beta^2\sin^2\phi_0}}$$

$$\cos\phi_\beta = \frac{\cos\phi_0}{\sqrt{1-\beta^2\sin^2\phi_0}}$$

[7] Stable means the relative positions of the rays determining the angle do not change with time or are taken at the same specified rest frame time.

or written with cosine only

$$\cos\phi_\beta = \frac{\cos\phi_0}{\sqrt{1-\beta^2+\beta^2\cos^2\phi_0}} \tag{35}$$

and reverse,

$$\cos\phi_0 = \frac{\Delta x_0}{d_0} = \frac{\cos\phi_\beta}{\gamma\sqrt{1-\beta^2\cos^2\phi_\beta}} \tag{36}$$

For angles ψ with both rays perpendicular to the x-axis the relationship is an identity, $\psi_\beta = \psi_0$.

Any angle with rays $\overrightarrow{j}$ and $\overrightarrow{k}$ co-planar with the x-axis equals the difference of the two angles formed with the x-axis by rays $\overrightarrow{j}$ and $\overrightarrow{k}$ respectively.

For any other angle with rays $\overrightarrow{j}$ and $\overrightarrow{k}$, conversion is possible either by transforming points and recomputing angles or by employing spherical trigonometry with the angle ψ between the two planes containing angles ϕ made by $\overrightarrow{j}$ and $\overrightarrow{k}$, respectively, with the x-axis.

Angles determined by non-simultaneous events, in contrast to stable angles determined by relatively fixed points, are especially frame dependent. Angles determined in a different frame by the same non-simultaneous events may be found by transformation of the determining events and applying the cosine law. In figure 1, for example, the angle with the x-axis of the light path from emission to reflection is $\angle CAB$ in the rest frame and $\angle CDB$ in the proper frame of P and Q.

3 Coming Attractions, (tentative)

3.1 Part Two tentative topics in preparation

Elementary Applications: Radar, Stellar Aberration, Small Angle Transformation, Loop and Polygonal Light Paths, The Sagnac Effect, Velocity Transformation, Relative Velocity of Two Objects, Inverse Frame Velocity, Composition of Velocities, Length Standardization and the Physics of Length Contraction,

3.2 Part Three tentative topics

The Experiment of Michelson and Morley, Maxwell's Electromagnetic Theory, An Empirical Method for Determining Absolute Velocity, The Theory of Mass, Inertia and Gravity, Orbital Dynamics, Particle Physics, The Theory of Atomic and Molecular Spectra.